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Reply on the Reply on the Note: "Scaling Transformations for Boundary Layer Flow near the Stagnation-Point on a Heated Permeable Stretching Surface in a Porous Medium Saturated with a Nanofluid and Heat Generation/Absorption Effects"

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The vehement reaction of Pop (2011) on my (Magyari 2011) Note concerning the recent paper by Hamad and Pop (2010) is unfounded in its content and inappropriate in its tone. It is also surprising since the main arguments listed in Pop (2011) against my paper (Magyari 2011) are simply unsustainable. Thus, below the equations

$$\alpha_{\mathrm{nf}} = \frac{k_{\mathrm{nf}}}{\left(\rho C_{\mathrm{p}}\right)_{\mathrm{nf}}},\ \rho_{\mathrm{nf}} = (1-\phi)\,\rho_{\mathrm{f}} + \phi\rho_{\mathrm{s}},\ \mu_{\mathrm{nf}} = \frac{\mu_{\mathrm{f}}}{(1-\phi)^{2.5}},$$
$$\left(\rho C_{\mathrm{p}}\right)_{\mathrm{nf}} = (1-\phi)\left(\rho C_{\mathrm{p}}\right)_{\mathrm{f}} + \phi\left(\rho C_{\mathrm{p}}\right)_{\mathrm{s}},\ \frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} = \frac{k_{\mathrm{s}} + 2k_{\mathrm{f}} - 2\phi\,(k_{\mathrm{s}} - k_{\mathrm{f}})}{k_{\mathrm{s}} + 2k_{\mathrm{f}} + \phi\,(k_{\mathrm{s}} - k_{\mathrm{f}})}, \tag{1}$$

of Pop (2011) (which are identical to Eq. 5 of Hamad and Pop 2010), a series of false assertions have been adduced, namely (citation):

> "The following book and the review papers (…) clearly demonstrate that in any theoretical and experimental work on nanofluid the physical properties should be taken into account. However, in the algebraic (mathematical) manipulations of Magyari's comments, these physical properties are hidden (scaled out). In fact, his new parameters, such as, λ, λ_1, Pr_1 and the new variable ξ should contain the *solid volume fraction of the nanofluid parameter* φ, that is $\lambda = \lambda(\varphi)$, $\lambda_1 = \lambda_1(\varphi)$, $Pr_1(\xi)$ and $\xi = \xi(\varphi)$, such that φ varies in the range $0 \leq \varphi \leq 0.2$ and the Prandtl number Pr for the base water fluid is $Pr = 6.8$. However, Magyari's parameters are not connected to the parameter φ, so that his problem is an ARTIFICIAL one and differs substantially by that of Hamad and Pop (2010) having nothing to do with nanofluid theory."

The first sentence of the above citation is trivial. Not only in nanofluid studies, but in every investigation of physical phenomena, ultimately the true physical properties should be taken into account. Furthermore, the subsequent three sentences of the above citation are totally false. First of all, the scaling transformations and the associated dimensionless parameter

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groups are not mere "algebraic manipulations", but basic concepts of fluid mechanics and of convective heat transfer, with a very deep physical significance. The most familiar example in this sense is the Reynolds number $Re = UL/\nu$ obtained from the transformations of the length and velocity scales. This dimensionless group tells us that for a given geometry and the same value of Re, the flow patterns of *all* viscous fluids are the same. Similarly, the dimensionless groups $\tilde{K} = K_1/\gamma$, $\tilde{P}r = Pr_1/\gamma$, λ_1 and $\tilde{S} = S\sqrt{\gamma}$ occurring in the rescaled equations (35)–(37) of my paper (Magyari 2011),

$$\dddot{f} + \tilde{K}\left(\frac{a}{c} - \dot{f}\right) + f\ddot{f} - \dot{f}^2 + \frac{a^2}{c^2} = 0 \tag{2}$$

$$\frac{1}{\tilde{P}r}\ddot{\theta} + f\dot{\theta} + \lambda_1\theta = 0 \tag{3}$$

$$\begin{aligned} &f(0) = \tilde{S},\ \ \dot{f}(0) = 1,\ \ \dot{f}(\infty) = a/c, \\ &\theta(0) = 1, \theta(\infty) = 0 \end{aligned} \tag{4}$$

lead us to the important physical insight that for a given value of the velocity ratio a/c the flow and heat transfer characteristics of *all fluids* are identical for the same values of the parameter set $\left(\tilde{K}, \tilde{P}r, \lambda_1, \tilde{S}\right)$. This holds both for regular fluids and the "nanofluid model" considered by Hamad and Pop (2010). Nothing is "hidden" in my approach and, contrary to Pop's assertion, the dependence on the volume fraction φ of the nanoparticles (denoted both in Magyari (2011) and Hamad and Pop (2010) by ϕ, instead of φ used in Pop 2011) is markedly present in Magyari (2011). Indeed, bearing in mind the above Eq. 1, as well as the definitions of Pr_1, λ_1 and γ given by Eqs. 12 and 23 of Magyari (2011),

$$Pr_1 = \frac{\alpha_f}{\alpha_{nf}}Pr,\ \lambda_1 = \lambda\left(1 - \varphi + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)^{-1},\ \gamma = (1-\varphi)^{2.5}\left(1 - \varphi + \varphi\frac{\rho_s}{\rho_f}\right) \tag{5}$$

one immediately arrives to the following explicit expressions of $\tilde{K}$, $\tilde{S}$ and $\tilde{P}r$

$$\tilde{K} = \frac{K_1}{\gamma} = \frac{K_1}{(1-\varphi)^{2.5}\left(1 - \varphi + \varphi\frac{\rho_s}{\rho_f}\right)},\ \tilde{S} = S\sqrt{\gamma} = S\sqrt{(1-\varphi)^{2.5}\left(1 - \varphi + \varphi\frac{\rho_s}{\rho_f}\right)} \tag{6}$$

and

$$\tilde{P}r = \frac{Pr_1}{\gamma} = \frac{\alpha_f}{\gamma\alpha_{nf}}\ Pr = \frac{1}{\gamma}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}\frac{k_f}{k_{nf}}Pr \tag{7a}$$

or, in more detail,

$$\tilde{P}r = \frac{1 - \varphi + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}}{(1-\varphi)^{2.5}\left(1 - \varphi + \varphi\frac{\rho_s}{\rho_f}\right)}\frac{k_s + 2k_f + \varphi(k_s - k_f)}{k_s + 2k_f - 2\varphi(k_s - k_f)}Pr \tag{7b}$$

The explicit dependence of the parameters $\left(\tilde{K}, \tilde{P}r, \lambda_1, \tilde{S}\right)$ on the volume fraction φ of the nanoparticles and on the other physical constants occurring in Eqs. 5–7a, 7b, emphasizes the absurdity of Pop's assertions formulated in the third and fourth sentence of the above citation. Pop's statement that my rescaled similarity variable ξ would not be connected to φ is erroneous, too. Indeed, the rescaled similarity variables (f, ξ) have been defined in terms of the original variables (F, η) of Hamad and Pop (2010) by Eq. 34 of Magyari (2011), i.e., $f(\xi) = \sqrt{\gamma}F(\eta), \xi = \sqrt{\gamma}\eta$. In this way, ξ is connected to φ directly,

$$\xi = \sqrt{(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)}\,\eta \tag{8}$$

Accordingly, the dimensionless surface shear stress $F''(0)$ and the surface temperature gradient $\theta'(0)$ occurring in the expressions of the friction coefficient C_f and of the local Nusselt number Nu_x given by Eq. 27 of Hamad and Pop (2010), can simply be obtained from their rescaled counterparts $\ddot{f}(0)$ and $\dot{\theta}(0)$ without any additional "research" effort as $F''(0) = \sqrt{\gamma}\ddot{f}(0)$, $\theta'(0) = \sqrt{\gamma}\dot{\theta}(0)$ that is

$$F''(0) = \sqrt{(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)}\ddot{f}(0), \theta'(0) = \sqrt{(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)}\dot{\theta}(0) \tag{9}$$

Furthermore, contrary to Pop's relationship $\lambda = \lambda(\varphi)$ given in the third sentence of the citation, the heat source/sink parameter λ defined in the first row below Eq. 10 of Hamad and Pop (2010) as $\lambda = Q_0/\left(\rho_f c C_p\right)$, definitely does not depend on φ. Thus, with $\lambda = \lambda(\varphi)$, Pop (2011) contradicts his own paper.

Therefore, nothing is "artificial" in the approach reported in Magyari (2011). Just the opposite is true. The rescaled boundary value problem is closely related to the "nanofluid model" considered in Hamad and Pop (2010), uncovering it as a "pseudo-nanofluid model", which is equivalent to the corresponding regular fluid model. To solve this pseudo-nanofluid model on the one hand, and the corresponding regular fluid model on the other hand, is exactly the same (numerical and analytical) task. The only (trivial) difference in the solution procedure consists in the numerical values which have to be taken for the rescaled parameters $\left(\tilde{K}, \tilde{P}r, \lambda_1, \tilde{S}\right)$ in the two mentioned cases. For the nanofluids, these values are obtained from the above Eqs. 5–7a,7b, while in the limiting case of vanishing volume fraction of the nanoparticles, $\varphi \to 0$, they reduce to the values (K_1, Pr, λ, S) of the corresponding (regular) base fluid. In other words, we are faced with one and the same problem with different numerical inputs. The aim of the two exactly solvable special cases reported in my paper (Magyari 2011) was to illustrate the above described scaling equivalence in a simple and convincing way. In this respect, the remark of Pop (2011) that "the effect of the parameter S is incomplete the Magyari's Note because he has not studied the cases of very large suction/injection parameter $|S| \gg 1$ as in other published papers" is not appropriate. Firstly, a short Comment is not a comprehensive review paper (the same holds also for the list of references) and secondly, Pop should actually know that an exact solution is valid for *all* (small, intermediary and large) values of the parameters involved. This feature becomes immediately obvious by a simple inspection of the exact solution given by Eq. 15 of Magyari (2011),

$$\theta(\eta) = e^{\xi_0^2-\xi^2} = e^{-\frac{1}{2}Pr_1\left(\eta^2+2S\eta\right)},\ \theta'(0) = -Pr_1 S \tag{10}$$

(For a second simple example, see also Eqs. 24 and 25 of Magyari (2011)).

Actually, my motivation to write the Note (Magyari 2011) was twofold. First, the non-dimensionalization chosen by Hamad and Pop (2010) raises the false impression that the governing equations of their "nanofluid model" (i.e., Eqs. 23 and 24 of Hamad and Pop 2010) differ essentially from the corresponding equations of a regular fluid, and thus, in spite of the neglected velocity-slip effects, it could possess some new physical content (which actually is not the case). My second reason was that similar pseudo-nanofluid models, which in fact all are scaling-equivalent to the corresponding regular fluid models, have recently been promoted by several other papers published elsewhere (Ahmad et al. 2010; Yacob et al. 2010, 2011). In this way, all the above explanations motivate and validate the main conclusion of my paper (Magyari 2011) once more: “The convective nanofluid heat transfer models which do not include the two main velocity slip effects of the nanoparticles with respect to the base fluid, namely the *Brownian diffusion* and the *thermophoretic diffusion*, are essentially equivalent to the corresponding viscous flow models for the base fluid”.

Bearing in mind the above arguments, the two last paragraphs of the reply (Pop 2011) do not deserve any answer. They only show that, paradoxically, the perception of Pop about the scaling transformations as “algebraic manipulations”, about the physical and “fictitious” effects, the pseudo and realistic nanofluid models, the original ideas and camouflaged trivialities and some more, is quite confusing. It is a bitter personal experience for me to be constrained by the offending reply (Pop 2011) of my old friend Ioan Pop on my objective paper Magyari (2011), to give him the above rebuttal on the pages of this Journal.

References

Ahmad, S., Rhoni, A.M., Pop, I.: Blasius and Sakiadis problems in nanofluids. Acta Mech. (2010). doi:10.1007/s00707-010-0414-6

Hamad, M.A.A., Pop, I.: Reply to the paper: Note on the “Scaling transformations for boundary layer flow near the stagnation-point on a heated permeable stretching surface in a porous medium saturated with a nanofluid and heat generation/absorption effects”. Transp. Porous Med. (2010). doi:10.1007/s11242-010-9683-8

Magyari, E.: Note on the “Scaling transformations for boundary layer flow near the stagnation-point on a heated permeable stretching surface in a porous medium saturated with a nanofluid and heat generation/absorption effects”. Transp. Porous Med. (2011). doi:10.1007/s11242-010-9712-7

Pop, I.: Reply to the paper TIPM1512: Note on the “Scaling transformations for boundary layer flow near the stagnation-point on a heated permeable stretching surface in a porous medium saturated with a nanofluid and heat generation/absorption effects”. Transp. Porous Med. doi:10.1007/s11242-011-9715-z

Yacob, N.A, Ishak, A., Nazar, R., Pop,I: Falkner–Skan problem for a static and moving wedge with prescribed surface heat flux in a nanofluid. Int. Comm. Heat Mass Transf. (2010) doi:10.1016/j.icheatmasstransfer.2010.12.003

Yacob, N.A, Ishak, A., Pop, I: Falkner-Skan problem for static or moving wedge in nanofluids. Int. J. Therm. Sci. **50**, 133–137 (2011)